NUCLEAR β- AND γ- COLLECTIVE BANDS IN THE $SU_q(2)$ ROTATOR MODEL

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ABSTRACT

The $SU_q(2)$ rotator model is used for describing the β_1- and γ_1-bands of even-even rare earth and actinide collective nuclei. Good results are obtained in nuclei with valence pair number $N > 10$. It is shown that in the excited bands the violation of the exact SU(2) symmetry is generally stronger than in the ground state bands, indicating the presence of a nonadiabatic perturbation caused by the excited vibrational degrees of freedom. The physical content of the parameter q is discussed. Predictions of the $SU_q(2)$ model for B(E2) intraband transitions in excited bands are presented and the need for specific experimental data is pointed out.

1. Introduction

The quantum algebra $SU_q(2)$[1,2] is a nonlinear generalization (having the structure of a Hopf algebra[3]) of the corresponding Lie algebra SU(2) to which it reduces when the deformation parameter q is set equal to one. It has been found that the $SU_q(2)$ algebra can be used for describing the deviations of rotational spectra of collective nuclei[4−7] and diatomic molecules[8−10] from the rigid rotator symmetry of SU(2), the deformation parameter q being related[5] to the softness parameter of the Variable Moment of Inertia (VMI) model[11]. Furthermore the implications of the $SU_q(2)$ symmetry on B(E2) transition probabilities within the ground state bands (gsb) of deformed nuclei have been considered[12], indicating that the B(E2) values do not saturate but continue to increase with increasing angular momentum I, a result also obtained in the framework of other models[13,14].

So far the $SU_q(2)$ symmetry has been tested only in relation to levels and B(E2) transition probabilities of the ground state band of deformed nuclei. It is the purpose of the present work to examine the applicability of the $SU_q(2)$ symmetry to excited collective bands, the β_1- and the γ_1-band in particular. Such an investigation is naturally motivated by the question: "It is already known that the quantum algebra $SU_q(2)$ is appropriate to characterize nuclear rotations built on the ground state, but what is the q-rotator in the case of a given excited band where besides

the rotational motion there is a presence of other collective (vibrational) degrees of freedom?" In this respect it is interesting to study whether the q-deformation "detects" the presence of the additional nonrotational degrees of freedom. As will be seen below, the study of the energy levels of the excited bands illuminates the above question and leads to interesting conclusions about the physical content of the deformation parameter q, while the study of the intraband B(E2) transition probabilities emphasizes the need for specific experimental data for testing the deviations from the pure SU(2) behavior appearing there.

2. q-rotator definition

The Hamiltonian of the q-rotator model is proportional to the second order Casimir operator $C_2[\mathrm{SU}_q(2)]$ of the quantum algebra $\mathrm{SU}_q(2)$[4]:

$$H = \frac{1}{2\theta} C_2[SU_q(2)] + E_0, \tag{1}$$

where θ is the moment of inertia parameter and E_0 is the bandhead energy (for the gsb $E_0 = 0$). The corresponding energy eigenvalues are:

$$E_I = \frac{1}{2\theta}[I][I+1] + E_0, \tag{2}$$

where I is the angular momentum and the square brackets indicate q-numbers, according to the following definition:

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}}. \tag{3}$$

In the case of q being a phase ($q = e^{i\tau}$ with τ a real parameter), eq. (2) gives:

$$E_I = \frac{1}{2\theta} \frac{\sin(\tau I)\sin\left(\tau(I+1)\right)}{\sin^2(\tau)} + E_0 \ . \tag{4}$$

In the limit $\tau \to 0$, the first term in eq. (4) gives the spectrum of the usual SU(2) rigid rotator[15]. It has been proved[5] that the deformation parameter τ is connected to the softness parameter of the VMI model, thus indicating that q-deformation is an alternative way of taking into account nuclear stretching.

3. $\mathrm{SU}_q(2)$ symmetry in exited collective bands

In the case of excited bands one needs an appropriately formulated q-rotator definition which should take into account the circumstance that the rotational energy levels are built on a given excited vibrational state[16]. For this purpose it is convenient to use eq. (4) in the form:

$$\overline{E_I} = E_I - E_0(I_{bh}) = \frac{1}{2\theta} \frac{\sin(\tau I)\sin\left(\tau(I+1)\right)}{\sin^2(\tau)} \tag{5}$$

with $I > I_{bh}$, where it is supposed that the energy scale of collective rotations has its origin in the bandhead energy $E_0(I_{bh})$, and I_{bh} is the bandhead angular momentum, which is 0 for β-bands and 2 for γ-bands. Thus, after subtracting the bandhead energy we determine the rotational parts of the bandlevels. However, it is important to remark that the so obtained energies are still perturbed by the vibrational motion as far as even in the well deformed nuclei the collective rotations are not separated completely from the vibrational degrees of freedom[16]. Taking into account this nonadiabatic perturbation we suppose that in the excited bands the q-deformations of the SU(2) symmetry should be generally larger in magnitude than the corresponding ones obtained in the gsb's. Hence one could expect that the quantum algebraic parameter τ will be able to indicate the presence of excited vibrational modes. Below it will be seen that the calculations essentially support this supposition.

So, the theoretical predictions (eq. 5) are compared to the experimental quantities $\overline{E_I^{exp}} = E_I^{exp} - E_0(I_{bh})$. For obtaining the fits an autoregularized iterational method of the Gauss-Newton type[17] has been used, the quality of the fits being measured by

$$\sigma = \sqrt{\frac{1}{n} \sum_{I=I_{min}}^{I_{max}} (\overline{E_I^{exp}} - \overline{E_I^{th}})^2}, \tag{6}$$

where n is the number of levels used in the fit and $I_{min} = 2$ for β-bands, while $I_{min} = 3$ for γ-bands. We have included in the fitting procedure rare earth and actinide nuclei in the rotational region (with $3 \leq R_4^g = E_4/E_2 \leq 10/3$) for which at least 5 levels of the β_1- or γ_1-band are known[18,19]. The results for the β_1- and/or γ_1-bands of 28 rare earths and 3 actinides, along with the results for the corresponding gsb are shown in Table 1.

The following comments can now be made:

i) The parameters τ_β and τ_γ generally obtain values in the region 0.03–0.07, close to the typical τ_g values of 0.03–0.06 (see also [4–7]). Nevertheless it is clearly seen that for almost all considered nuclei, the τ values obtained in the excited bands lie above the corresponding gsb values (see also Fig. 1). It turns out that in the excited bands the quantum algebraic parameter τ, which characterizes the deviation of the spectrum from the pure SU(2) symmetry of the rigid rotator[20], indicates the presence of additional nonrotational degrees of freedom. Moreover, some $\tau_\gamma \geq 0.1$ values occur for nuclei with valence pair number N relatively small (10–13), indicating that the rotational character of the γ_1-band is not yet well developed in this N region. So, the sensitivity of the $SU_q(2)$ rotator description to the structure of the different types of bands is obvious.

Table 1. Parameters of the fits of β_1- and γ_1-bands in the rare earth and actinide regions using eq. (5). The deformation parameters τ_β and τ_γ, the quality factors σ_β and σ_γ (in keV) (eq. 6) accompanied by the numbers n_β and n_γ of the experimental levels used in the fit, and the inertial parameters $1/(2\theta_\beta)$ and $1/(2\theta_\gamma)$ (in keV^{-1}) for the β_1- and γ_1-bands respectively are shown. The corresponding deformation parameters τ_g of the ground state band and the valence pair numbers N are also given. The experimental data are taken from [18,19].

Nucleus	N	τ_g	τ_β	τ_γ	$\sigma_\beta[n_\beta]$	$\frac{1}{2\theta_\beta}$	$\sigma_\gamma[n_\gamma]$	$\frac{1}{2\theta_\gamma}$
^{152}Sm	10	0.0622	0.0695	0.1030	22.57[8]	15.39	15.13[8]	21.63
^{154}Sm	11	0.0500		0.1306			1.14[5]	18.67
^{156}Gd	12	0.0521	0.0641	0.0668	5.20[6]	12.36	11.29[10]	14.92
^{158}Gd	13	0.0419		0.1345			4.96[5]	14.87
^{160}Gd	14	0.0392		0.0507			0.31[5]	11.68
^{156}Dy	12	0.0733		0.0727			17.72[11]	18.66
^{160}Dy	14	0.0489		0.0715			0.71[5]	14.24
^{162}Dy	15	0.0368	0.0456	0.0339	7.76[8]	8.56	19.01[13]	12.07
^{164}Dy	16	0.0391		0.0672			3.40[5]	11.70
^{160}Er	12	0.0839		0.1158			4.76[5]	22.53
^{162}Er	13	0.0538		0.0605			12.19[11]	16.39
^{164}Er	14	0.0463		0.0531			19.94[13]	14.49
^{166}Er	15	0.0461	0.0932	0.0520	18.10[7]	12.47	5.97[13]	12.59
^{168}Er	16	0.0353	0.0400	0.0321	0.50[5]	9.79	0.07[7]	12.50
^{170}Er	17	0.0348		0.0438			10.11[6]	13.15
^{166}Yb	13	0.0610		0.0743			8.84[6]	17.18
^{168}Yb	14	0.0499		0.0674			2.20[6]	13.91
^{170}Yb	15	0.0428	0.0577	0.0342	6.49[7]	11.08	11.57[8]	13.13
^{172}Yb	16	0.0327	0.0584		7.75[8]	12.12		
^{172}Hf	14	0.0503		0.0687			9.94[10]	17.55
^{174}Hf	15	0.0496	0.0439		5.07[5]	11.76		
^{176}Hf	16	0.0449	0.0632	0.0673	3.79[6]	12.05	17.17[7]	16.26
^{178}Hf	15	0.0470		0.0867			1.90[5]	16.22
^{180}Hf	14	0.0357		0.0434			2.24[6]	15.15
^{178}W	15	0.0537	0.0545		6.23[8]	13.61		
^{180}W	14	0.0591		0.0883			8.20[7]	18.89
^{182}W	13	0.0607		0.1140			9.70[5]	18.93
^{184}W	12	0.0476		0.0681			1.10[5]	17.22
^{232}Th	12	0.0314	0.0378	0.0424	1.50[8]	7.07	4.80[13]	7.44
^{232}U	12	0.0364	0.0393		0.34[6]	7.15		
^{234}U	13	0.0295	0.0363	0.0514	0.45[5]	6.92	0.29[6]	7.12

ii) It is known[20] that for the ground state bands of the rare earths and the actinides the parameter τ_g decreases with increasing valence pair number N (or, equivalently, with increasing neutron valence pair number N_ν in a given group of isotopes) approximately as

$$\tau = \sqrt{3}(8N^2 + 22N - 15)^{-\frac{1}{2}} \ , \tag{7}$$

indicating that τ_g, as a measure of deviation from the rigid rotator symmetry, indirectly reflects the nuclear shell structure. We remark that the same trend is seen for the τ_γ values, especially in the case of the Er isotopes (shown in Fig. 1) and the Yb isotopes. In the excited bands it is difficult to derive an analytical relation between τ and N, but Fig. 1 clearly shows that such a correlation actually exists. We thus conclude that in the γ-bands the $\mathrm{SU}_q(2)$ symmetry quite well characterizes the deterioration of the nuclear rotational properties away from the midshells.

Fig. 1. Deformation parameters τ_g (circles, connected by solid lines) and τ_γ (triangles, connected by dashed lines) for ground state bands and γ_1-bands respectively of Er isotopes (taken from Table 1) are plotted versus the valence pair number N.

iii) we remark that the above behavior of the parameter τ_γ allows one to make some additional conclusions. It has been shown[21] that in the gsb's the correlation between τ and N given approximately by eq. (7) allows one to connect τ with the axial deformation parameter β:

$$\beta \sim \left(B/[3(2B + 60.25)^{1/2} - 22.5] \right)^{1/2} \ , \tag{8}$$

where $B = 1/(1 - \tau \cot \tau)$. Thus it has been obtained that β decreases with the increase of τ and that τ could be considered as a relevant measure of decrease in deformation as well as in rotational collectivity of the nuclei in a given rotational region. Though in the excited bands we have not such analytical estimates, Fig. 1 implies that in the γ-bands the decrease of τ_γ towards the midshells, could be associated similarly with the corresponding increase of nuclear deformation and rotational collectivity. In this case the relevance of the quantum algebraic approach is obvious. The data on β-bands are not enough for drawing any conclusions about the τ_β values.

4. B_q(E2) transitions in the exited bands

We now turn to the study of the B(E2) transition probabilities within β- or γ-bands. In the usual case the B(E2) values are given by

$$B(E2; I_i \to I_f) = \frac{5}{16\pi} {Q_0}^2 |C^{I_i,2,I_f}_{K,0,K}|^2, \tag{9}$$

where Q_0 is the intrinsic quadrupole moment and $C^{j_1,j_2,j}_{m_1,m_2,m}$ are the Clebsch-Gordan coefficients of the Lie algebra SU(2). In the case of $\mathrm{SU}_q(2)$ one should use the q-generalized angular momentum theory[22,23,24], in which the irreducible tensor operators for the quantum algebra $\mathrm{SU}_q(2)$[23] as well as the q-generalized version of the Wigner-Eckart theorem[24] are available. The q-deformed versions of the Clebsch-Gordan coefficients needed for the q-generalization of eq. (9),

$$B_q(E2; I_i \to I_f) = \frac{5}{16\pi} {Q_0}^2 |{}_qC^{I_i,2,I_f}_{K,0,K}|^2, \tag{10}$$

are also known[22,23,24]. In the case of intraband transitions with $\Delta I = I_i - I_f = 2$ one needs

$${}_qC^{I+2,2,I}_{K,0,K} = q^{-2K} \left(\frac{[3][4][I+K+1][I+K+2][I-K+2][I-K+1]}{[2][2I+2][2I+3][2I+4][2I+5]} \right)^{\frac{1}{2}}, \tag{11}$$

while in cases with $\Delta I = 1$

$${}_qC^{I+1,2,I}_{K,0,K} = -q^{I-2K+2}([I+K]-q^{2I}[I-K]) \left(\frac{[2][3][I+K+1][I-K+1]}{[2I][2I+2][2I+3][2I+4]} \right)^{\frac{1}{2}} \tag{12}$$

is needed, where the square brackets again indicate q-numbers as defined in eq. (3) with $q = e^{i\tau}$.

Therefore in the case of β-bands ($K = 0$) one finds

$$B_q(E2; I+2 \to I) = \frac{5}{16\pi} {Q_0}^2 \frac{[3][4][I+1]^2[I+2]^2}{[2][2I+2][2I+3][2I+4][2I+5]} \ . \tag{13}$$

In the case of γ-bands ($K = 2$) for $\Delta I = 2$ transitions one has

$$B_q(E2; I+2 \to I) = \frac{5}{16\pi} {Q_0}^2 \frac{[3][4][I-1][I][I+3][I+4]}{[2][2I+2][2I+3][2I+4][2I+5]} \ , \qquad (14)$$

while for $\Delta I = 1$ transitions one finds

$$B_q(E2; I+1 \to I) = \frac{5}{16\pi} {Q_0}^2 \left([I+2]^2 + [I-2]^2 - 2\cos(2\tau I)[I-2][I+2] \right)$$

$$\frac{[2][3][I+3][I-1]}{[2I][2I+2][2I+3][2I+4]}. \qquad (15)$$

Fig. 2. $B_q(E2; I+2 \to I)$ transition probabilities are plotted as a function of angular momentum I in the cases of β-bands (eq. 13, solid lines) and γ-bands (eq. 14, dashed lines) for some typical values of the deformation parameter τ. The numerical values of $B_q(E2)$ correspond to $\frac{5}{16\pi} Q_0^2 = 1$. The limiting case $\tau = 0$ gives the usual rigid rotator predictions.

On these results the following comments apply:

i) eq. (13), concerning the β-bands, is exactly the same as the one obtained in the case of gsb[4,12]. It has been shown that this equation gives B(E2) values increasing with increasing I, while the corresponding usual SU(2) expression (obtained here for $\tau \to 0$) exhibits saturation with increasing I. This is illustrated in Fig. 2. In the case of gsb's some experimental examples supporting this prediction have been given in [12]. Similar predictions also occur in the framework of other models[13,14]. The existing data for β_1-bands do not suffice for testing this prediction.

ii) eq. (14), concerning $\Delta I = 2$ transitions in γ-bands, gives almost the same behavior as eq. (13), as seen in Fig. 2. It follows that for $\Delta I = 2$ transitions the introduction of q-generalized Clebsch-Gordan coefficients leads to a typical modification of the reduced transition probabilities in all considered bands.

Fig. 3. Same as Fig. 2 but for the case of $B_q(E2; I+1 \to I)$ transition probabilities in γ-bands.

iii) eq. (15), concerning $\Delta I = 1$ transitions in γ-bands, illustrated in Fig. 3, gives an interesting prediction. For typical τ-values (0.03–0.07) one initially observes a decrease of $B_q(E2; I+1 \to I)$ with increasing I, but further, after reaching some minimum (for example at $I = 5$ when $\tau = 0.05$), a significant increase of $B_q(E2)$ is observed, while in the rigid rotator limit ($\tau \to 0$) a continuous decrease down to zero at sufficiently large $I > 12$ is predicted. The available data for $E2$ intraband transitions in the excited bands do not suffice for detailed tests of these predictions, due to the short life times and strong $M1$ mixing observed in these transitions. The need for further experimental data is clear. In particular the observation of any $E2$ transitions with $\Delta I = 1$ at $I > 10 - 12$ in the γ-bands will be useful in testing the predictions of eq. (15).

We now remark that the present investigation outlines the principal limits of the $SU_q(2)$-symmetry approach to the nuclear rotational spectra. It should be emphasized that in the framework of the quantum algebra $SU_q(2)$ as well as in the case of the standard Lie algebra $SU(2)$[25], one is able to provide a consistent description of the physical characteristics of only one given rotational band. This is clearly indicated by the distinctions in the magnitudes of the q-deformation parameter obtained for the different types of bands (see Table 1). It follows that one

should understand the $SU_q(2)$-rotator as a one-band model based on the particular intrinsic state or vibrational mode. Hence the unified description of the different rotational bands including the calculation of the interband transition probabilities is beyond the limits of the quantum algebra $SU_q(2)$. Such an extension could be referred to a model based on the q-deformed algebra $SU_q(3)$ in which the introduction of a bandmixing interaction would be possible. However the realization of such a model is still complicated due to some difficulties in the obtaining of the reduction $SU_q(3) \supset SO_q(3)$ (for example see [26,27]). In this respect the use of the simplest quantum algebra $SU_q(2)$ could be considered as a first approximation in the construction of a more complicated quantum algebraic theory of nuclear collective motion.

5. Conclusion

In conclusion, we have demonstrated the relevance of the $SU_q(2)$ approach beyond the ground state bands, namely in the excited bands of even-even rare earth and actinide nuclei. Good results have been obtained for β_1 and γ_1 bands in nuclei with valence pair number $N > 10$. The quantum algebraic parameter τ fitted in these bands obtains values generally shifted above the corresponding ones in the gsb's. In such a way the q-deformation specifically indicates the presence of a nonadiabatic perturbation caused by the excited vibrational degrees of freedom. The decrease of τ_γ and τ_g with increasing N is in accordance with the interpretation of τ as a measure of deviation from the rigid rotator limit equivalent to the nuclear softness[5,20]. In addition, these correlations (illustrated in Fig. 1) allow one to extend the $SU_q(2)$ symmetry to a wider range of nuclear rotational properties[21]. The predictions of the $SU_q(2)$ rotator model for the $B(E2)$ intraband transition probabilities in the excited bands show modifications in comparison to the SU(2) rigid rotator limit, the experimental data needed for testing these predictions having been identified. It is pointed out that $SU_q(2)$ is a simple one-band model, but it can be considered as the first step in the development of more complicated models based on the q-deformed algebras.

6. References

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